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What Do We Learn about Quantum Mechanics from the Theory of Measurement?

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It is argued that a quantum mechanical analysis of the measurement process permits one to adjudicate between an individual system interpretation of the state vector and an ensemble interpretation, in favor of the latter. Possible changes to quantum mechanics that would be necessary to enable it to describe individual systems are discussed.

1. INTRODUCTION

The quantum theory of measurement (QTM) has now developed to the point where it may be applied to the design of sensitive measurements. However, the subject of this paper is a much older use of it, in which an empirical and intuitive understanding of measurement is used to test, or otherwise illuminate, the interpretation of quantum mechanics (QM). The individual components of this paper are not new, but it seems to me that the conclusions which necessarily follow from them are not as well known as are the separate components. These conclusions significantly affect the way in which QM can and should develop.

2. TWO INTERPRETATIONS OF THE QM "STATE"

A central and essential issue in the interpretation of QM is the nature of the "state" concept. The major positions on this issue were defined long ago, and they divide naturally into two classes:

(A) A pure state $|\Psi\rangle$ provides a complete and exhaustive description of an *individual* system. A dynamical variable represented by the

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operator Q has a value (q, say) *if and only if* $Q|\Psi\rangle = q|\Psi\rangle$; otherwise the variable is undefined (not merely unknown, but *nonexistent*).

One may distinguish variants of this view: an *objective* version in which $|\Psi\rangle$ is taken to be a kind of physical property of the system, and a *subjective* version in which $|\Psi\rangle$ represents some observer's knowledge of the system.

(B) A pure (or mixed) state describes the statistical properties of an *ensemble* of similarly prepared systems.

In this view $|\Psi\rangle$ is not itself an element of reality, and the significance of the "state" concept is only as a collection of probabilities. A well-defined state preparation procedure yields a well-defined probability distribution for each observable of the system. Two variants may be identified, each having to do with the interpretation of probability. If one adopts a *frequency* interpretation of probability, then one will associate the "state" concept with the potential ensemble of systems that may result from repetitions of the state preparation procedure. If one adopts a *propensity* interpretation of probability (Popper, 1957), then one will associate the probabilities (and hence the "state") with the physical conditions and repeatable procedures that generate the ensemble, rather than with the ensemble itself. This difference will not be important in this paper.

Interpretation (A) was dominant throughout the early history of QM, and is taken for granted in many textbooks. However, I shall argue that the conclusion to be drawn from the QTM is that interpretation (A) is untenable. A systematic exposition of interpretation (B) was given by Ballentine (1970). For purposes of this paper it should not be regarded as a unique interpretation, but rather as a class of interpretations consistent with the statement (B) above.

3. ANALYSIS OF MEASUREMENT

The process of measurement involves an *object* (I) and an *apparatus* (II). We wish to measure some dynamical variable R belonging to the object. The operator corresponding to R has a complete set of eigenvectors,

$$R|r\rangle_{\mathrm{I}} = r|r\rangle_{\mathrm{I}} \tag{1}$$

The measurement apparatus (II) has an indicator variable A and a corresponding complete set of eigenvectors,

$$A|\alpha, m\rangle_{\mathrm{II}} = \alpha|\alpha, m\rangle_{\mathrm{II}} \tag{2}$$

Here α is the "indicator position" eigenvalue, and m labels all the many other quantum numbers that are needed to specify a unique eigenvector.

Let the apparatus be prepared in an initial premeasurement state $|0, m\rangle_{\text{II}}$, with $\alpha = 0$. An interaction between the object and the apparatus is then introduced, which must be designed to produce a unique correlation $r \to \alpha_r$ between the initial value of r and the final value of $\alpha = \alpha_r$. The necessary properties of the interaction are specified implicitly by placing appropriate constraints on the evolution operator U. For so-called "ideal" measurements, which do not change the state of the object, the appropriate condition is

$$U|r\rangle_{\text{I}} \otimes |0, m\rangle_{\text{II}} = |r\rangle_{\text{I}} \otimes |\alpha_r, m'\rangle_{\text{II}} \tag{3a}$$

This "ideal" case is unrealistic for most measurements, and is unnecessarily restrictive for theoretical purposes. The most general necessary condition on U is

$$\begin{aligned} U|r\rangle_{\text{I}} \otimes |0, m\rangle_{\text{II}} &= \sum_{r',m'} u^{r,m}_{r',m'} |r'\rangle_{\text{I}} \otimes |\alpha_r, m'\rangle_{\text{II}} \\ &= |\alpha_r; (r, m)\rangle, \quad \text{say} \end{aligned} \tag{3b}$$

The final-state vector of (3b) is labeled by the "indicator position" eigenvalue α_r. The other labels (r, m) are not eigenvalues, but merely labels indicating where this vector came from via the unitary transformation U. (Unitarity will also impose some restrictions on the matrix u, but these need not be spelled out.) The general case (3b) imposes on the measurement interaction only one essential requirement: that it establish the correspondence $r \to \alpha_r$. The values of α_r corresponding to different values of r should be clearly distinguishable by eye, and so I shall refer to them as *macroscopically distinct* values.

If the initial state of the object is not an eigenstate of the variable R being measured, but rather is of the form

$$|\Psi\rangle_{\text{I}} = \sum_r c_r |r\rangle_{\text{I}} \tag{4}$$

then from (3b) and the linearity of the evolution operator U we obtain

$$\begin{aligned} U|\Psi\rangle_{\text{I}} \otimes |0, m\rangle_{\text{II}} &= \sum_r c_r |\alpha_r; (r, m)\rangle \\ &= |\Psi^f_m\rangle, \quad \text{say} \end{aligned} \tag{5}$$

This result is important enough to be stated as a theorem, even though we later need to extend its proof to more general situations.

Measurement Theorem. The final state of the entire system (object + apparatus) is, in general, a *coherent superposition* of *macroscopically distinct* "indicator position" eigenvectors.

The final state (5) does not satisfy the eigenvalue equation (2), so according to interpretation (A), the indicator position of the apparatus should be undefined after the measurement. But this is contrary to observation, and therefore in order to save interpretation (A) one must introduce the following:

Projection Postulate. The final-state vector is somehow "reduced" from the coherent superposition state $|\Psi_m^f\rangle$ obtained in (5) to either (i) an incoherent mixture of "indicator position" eigenvectors or (ii) a single eigenvector $|\alpha_{ro};(r_0, m)\rangle$, where α_{ro} is the observed value of the "indicator position."

Both versions of the Projection Postulate are apparently incompatible with the Schrödinger equation of motion for the entire system; however, there have been a number of attempts to reconcile the two. Some of those attempts, and their refutations, are listed below.

(a) The "reduction" is caused by an unpredictable and uncontrollable disturbance of the object by the measuring apparatus.[2]

If the interaction between the object and the apparatus satisfies the minimal necessary condition (3b), then that interaction will *cause*, rather than destroy, the coherent superposition.

(b) The observer causes the "reduction" upon reading the result of the measurement from the apparatus.[3]

The strangely psychic character of this proposal can be eliminated simply by including both the observer and the apparatus in the definition of II, whereupon the Measurement Theorem applies as in (a).

(c) The action of the environment upon the system causes the "reduction" of the state.[4]

Any portion of the environment whose effect is expected to be significant should be included with the apparatus in the definition of part II of the

[2]The "disturbance" theory is the oldest attempt to explain the "state reduction" process, its probable origin being in Heisenberg's "x-ray microscope" argument. It was abandoned by Bohr in 1935 following the Einstein-Podolsky-Rosen paper, and is no longer widely advocated. Yet a book as recent as Messiah (1964, p. 140) still used it.

[3]This view appears to be advocated by Wigner (1961). He has indicated that he does not regard it as true, but rather as an apparent consequence of the theory, showing (in the manner of a *reductio ad absurdum* argument) that something must be wrong with the orthodox theory.

[4]This idea is expressed more positively by Zurek (1986). In fairness, it should be pointed out that he does not claim that the state of the total system (object + apparatus + environment) will achieve the desired form of equation (7), but only that the partial state obtained by tracing over certain environmental variables will do so. However, I claim that even if his result were conclusively proven, which is not yet the case, it would not be relevant to the question of interpretation of QM that is the subject of this paper. The partial state operator for one part of a two-component system, obtained by tracing over the variables of the other part, will omit all correlations between the two parts. It is therefore not an appropriate tool for studying fundamental questions of interpretation, particularly in view of the fact that the division of the system into two parts is arbitrary and conventional. Such questions can only be answered by considering the total state of the system, including any relevant portions of the environment.

system. The coordinates of the environment then become part of the set labeled "m" in (2). The Measurement Theorem then applies without modification.

(d) The initial state of the apparatus II cannot, in practice, be specified as a unique pure state, as the notation $|0, m\rangle_{\text{II}}$ would imply, since the state preparation procedure will not reproduce the same "m" upon each repetition. (This is especially the case if large parts of the environment are included in the definition of II.) If the initial state is a mixed state, then the final state will also be a mixed state, apparently opening the possibility of saving version (i) of the Projection Postulate [though not version (ii)].[5]

This is the only proposal that requires a nontrivial response, although the conclusion is unaltered.

Instead of the initial pure state vector assumed in (5),

$$|\Psi^i_m\rangle = |\Psi\rangle_{\text{I}} \otimes |0, m\rangle_{\text{II}}$$

we now assume an initial mixed state for the system,

$$\rho^i = \sum_m w_m |\Psi^i_m\rangle\langle\Psi^i_m| \tag{6}$$

Here w_m can be regarded as the probability associated with each of the microscopic states labeled by m. The hope of an advocate of interpretation (A) would now be that the final state would be a mixture of "indicator position" eigenstates, perhaps of the form

$$\rho^d = \sum_r |c_r|^2 \sum_m v_m |\alpha_r; (r, m)\rangle\langle\alpha_r; (r, m)| \tag{7}$$

but certainly *diagonal* with respect to α_r, since any nondiagonal terms would correspond to coherent superpositions of "indicator position" eigenvectors.

That hope is unfounded. The actual final state is

$$\rho^f = U\rho^i U^{\dagger} = \sum_m w_m |\Psi^f_m\rangle\langle\Psi^f_m| \tag{8}$$

where $|\Psi^f_m\rangle = U|\Psi^i_m\rangle$. From (5) we obtain

$$\rho^f = \sum_{r1}\sum_{r2} c^*_{r1} c_{r2} \sum_m w_m |\alpha_{r1}; (r_1, m)\rangle\langle\alpha_{r2}; (r_2, m)| \tag{9}$$

The terms with $\alpha_{r1} \neq \alpha_{r2}$ indicate coherent superposition of macroscopically distinct "indicator position" eigenvectors, just as was the case in (5), and it is clear that these terms do not cancel out. Thus, the Measurement Theorem holds for a mixed initial state as well as for a pure initial state.

[5]This seems to have been the view of Heisenberg (1958, pp. 53–55).

The proof can also be extended to approximate measurements (Shimony, 1974). The suggestion that the problems of measurement and irreversibility might be closely connected has been discussed elsewhere (Ballentine, 1986). Strictly speaking, any irreversible processes that may occur are already included in (c) and (d), so the matter will not be considered further in this paper.

4. WHAT HAVE WE LEARNED?

The conclusion of this analysis, expressed in the Measurement Theorem, is that a kind of interaction that is not at all exotic will lead inevitably to a *coherent superposition* of terms that together describe *macroscopically distinct* values of a certain quantity (the "indicator position" of the apparatus). No amount of mathematical sophistication or detailed calculation on more realistic models can alter this fact, anymore than they could alter the theorem of energy conservation. If this sort of coherent superposition is unacceptable within any interpretation, as is apparently the case with interpretation (A), then *that interpretation is untenable and must be abandoned.*

The "measurement problem," viewed as the problem of explaining how the "reduction of the state vector" comes about, is insoluble. There is no such physical process as "state reduction," at least not as long as the accepted mathematical formalism of QM is correct. On the other hand, the "measurement problem" viewed as seeking an interpretation of the formalism that is compatible with the Measurement Theorem is solved by adopting an *ensemble* interpretation of the state vector, since interpretation (B) does not require the nonexistent "state reduction" process.

5. WHERE DOES IT LEAD US?

Since, as has been argued, the QM state vector describes only an ensemble of similarly prepared systems, then there is a need for a theory that does describe individual systems. This need is especially felt in cosmology, where there is no room for any observer or environment outside of the system, and where probabilistic predictions of the kind that serve so well in atomic physics are untestable because it is not possible to perform measurements on an ensemble of similarly prepared universes.

It must be emphasized that a description of individual phenomena cannot be obtained within QM merely by a change of interpretation. Any interpretation, old or new, that incorporates the tenets of (A) will encounter the insoluble "measurement problem." There are two possible routes to a description of individual systems: one may add additional structure to QM compatible with the existing formalism; or one may modify the formalism in some way.

A good example of a structure added within the existing framework of the statistical quantum theory is the quantum potential of Bohm (1952), which generates individual particle trajectories that automatically reproduce the usual position probability density $|\Psi(\mathbf{r})|^2$. These trajectories have now been computed in detail for several situations involving interference (Philippidis *et al.*, 1979; Dewdney *et al.*, 1987). The quantum potential is determined from the wavefunction, $V_Q(\mathbf{r}) = (-\hbar^2/2m)\,\nabla^2 R/R$, where $R = |\Psi|$. At first sight this approach seems to go against the spirit of the statistical interpretation (B), since Ψ is no longer merely a generator of probabilities for the ensemble. Now Ψ becomes an element of physical reality in the individual case through its connection with V_Q. But the fatal flaw of interpretation (A) is avoided by denying its second tenet: according to Bohm's theory, the position of a particle always has a definite value, even though Ψ need not resemble a position eigenfunction. Bohm's theory is of value because it provides a model of the individual phenomena that are described only statistically by QM. The model is not likely to be unique, and generalizations should be sought.

The other possible route to the description of individual systems necessarily requires some modification of the mathematical formalism, and the number of possibilities seems almost limitless. One of them is to modify the Schrödinger equation so that coherent superpositions of macroscopically distinct states will quickly and spontaneously disappear, making it plausible to reinterpret the state vector Ψ as the description of an individual system. The rationale for this approach is described by Pearle (1986). It should be emphasized that such modifications of QM are *not* needed to solve any "problem of measurement." Rather, they are needed because without modification QM cannot provide a description of individual systems, the analysis of the measurement process being only a convenient means of demonstrating this fact. Most of these theories for "spontaneous reduction" of coherent superpositions are rather *ad hoc*, and hence not very compelling. A notable exception is the recent theory of Diosi (1987). He postulates that the gravitational field is not quantized, but is subject to fluctuations of the form and magnitude suggested by the Bohr–Rosenfeld analysis. Since gravity couples universally to the mass density of all systems, the theory has no arbitrary parameters. Assuming a white noise spectrum for the fluctuations, he derives a master equation for the evolution of the noise-averaged statistical operator. This equation permits "pure" states to evolve into "mixed" states as long-range coherence decays spontaneously. The maximum coherence length of a neutron is estimated to be 10^6 cm, but that of a colloidal grain of radius 10^{-5} cm is estimated to be only of the order of the radius itself. Thus, the familiar predictions of QM for atoms should not be significantly altered, but coherent macroscopic superpositions of the type predicted in the analysis of measurement should be suppressed. It will be interesting to devise experiments to test this theory in the regime intermediate between the atomic and macroscopic domains.

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